

Sweeping reciprocal vortex lattice across the Fermi surface: A new magnetoquantum oscillations effect in the superconducting state

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It is shown that coherent scatterings by an ordered vortex lattice are critically enhanced for quasi particles moving in cyclotron orbits on the Fermi surface through vortex core regions, thus generating significant quasi-periodic oscillating contributions to the SC free energy as a function of the inverse magnetic field. The mean frequency of the oscillation provides a fingerprint of the vortex lattice geometry. Vortex-lattice disorder, tends to suppress this oscillatory component.

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Many reports on observation of de Haas van Alphen (dHvA) or Shubnikov-de Haas (SdH) oscillations in the mixed (vortex) states of strongly type-II superconductors, have shown additional damping of the oscillations' amplitude in the vortex state with respect to the normal state signal (see, e.g.¹⁻⁶). In all these experiments the appearance of the extra damping has been found structureless, reflecting only some average effect of the vortex matter on the fermionic quasi particles. Extracting valuable information about the vortex state from these data requires development of a comprehensive quantitative theory of pure strongly type-II superconductors at high magnetic fields and low temperatures, a theory which does not exist today even within the conventional BCS framework⁷. Recently, using a theoretical approach based on an exact perturbative Gorkov-Ginzburg-Landau theory⁸, it was found, however, that on certain cyclotron orbits fermionic quasi particles (QPs) are singularly coupled to, and coherently scattered by the vortex lattice, resulting in a new type of magnetoquantum oscillations superimposed on the usual dHvA oscillations. In the present paper we analyze characteristic features of the discovered oscillatory effect and discuss their experimental feasibility. The kinematical condition controlling the predicted effect is shown, by means of a simple model, to be operative beyond the limitations of the perturbation theory.

Following the theory presented in Ref.⁸ we consider a 2D strongly type-II (neglecting the effect of SC screening currents) superconductor in a perpendicular uniform magnetic field $\mathbf{H} = H\hat{z}$. Generalization to isotropic 3D systems is rather straightforward. It is assumed that the superconductor can be described by means of BCS-

Hamiltonian for the usual singlet *s*-wave electron pairing (electron spin is neglected for the sake of simplicity). Within mean-field approximation (but not in a fully self-consistent manner) the order parameter is described by

a general vortex lattice state, $\Delta(\mathbf{r}) = \left(\frac{2\pi}{a_x^2}\right)^{1/4} \Delta_0 \varphi_0(\mathbf{r})$, written in terms of a discrete set of ground-state Landau orbitals (in symmetric gauge):

$\varphi_0(x, y) = e^{ixy} \sum_n e^{-i\theta n^2 + iq_n x - (y + q_n/2)^2}$, where $q_n = \frac{2\pi}{a_x} n = q_0 n$, $n = 0, \pm 1, \pm 2, \dots$ with the lattice spacing a_x along the *x* - axis and $a_x^2 = \pi / \sqrt{1 - (\theta/\pi)^2}$. For the Abrikosov triangular and square lattices: $\theta = \pi/2$, and $\theta = 0$ respectively. We use dimensionless space coordinates measured in units of the magnetic length, $a_H = \sqrt{c\hbar/eH}$. The amplitude of the order parameter, $\Delta_0^2 = S^{-1} \int d^2\mathbf{r}_i |\Delta(\mathbf{r}_i)|^2$, where $S = \pi N$ and N is the number of vortices, is treated as a variational parameter with respect to the SC thermodynamic potential (TP) $\Omega_{sc}(\Delta_0)$, which can be written as a Taylor expansion in Δ_0 :

$$\Omega_{sc}(\Delta_0) = S \left(\frac{\Delta_0^2}{g_{BCS}} \right) + \sum_{n=1} \frac{(-1)^n}{n} \Omega_{2n}(\Delta_0) \quad (1)$$

where $\Omega_{2n}(\Delta_0) = N k_B T \left(\frac{\Delta_0}{\hbar \omega_c} \right)^{2n} I_{2n}$, g_{BCS} is the BCS coupling constant and $\omega_c = eH/mc$. The quartic term, I_4 , is the leading contribution to the SC free energy influenced by the vortex distribution⁷. An exact expression derived in⁸ for a given Matsubara frequency $\omega_\nu = \pi k_B T (2\nu + 1) / \hbar$, reads:

$$I_4 = \int_0^\infty d\tau_1 d\tau_2 d\tau_3 d\tau_4 e^{-\varpi_\nu(\tau_1 + \tau_2 + \tau_3 + \tau_4) - i n_F(\tau_1 - \tau_2 + \tau_3 - \tau_4)} \frac{\beta(\gamma)}{\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4} \quad (2)$$

where $\varpi_\nu = \omega_\nu / \omega_c$ and $n_F = E_F / \hbar \omega_c - 1/2$. Here $\beta(\gamma) = \sum_{\mathbf{G}} \beta_{\mathbf{G}}(\gamma)$ is a function of the 4-electron variable $\gamma = \frac{\alpha_2 \alpha_4 - \alpha_1 \alpha_3}{\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4}$, $\alpha_j = 1 - e^{i\varepsilon_j \tau_j}$, $\varepsilon_j = (-1)^{j+1}$, and:

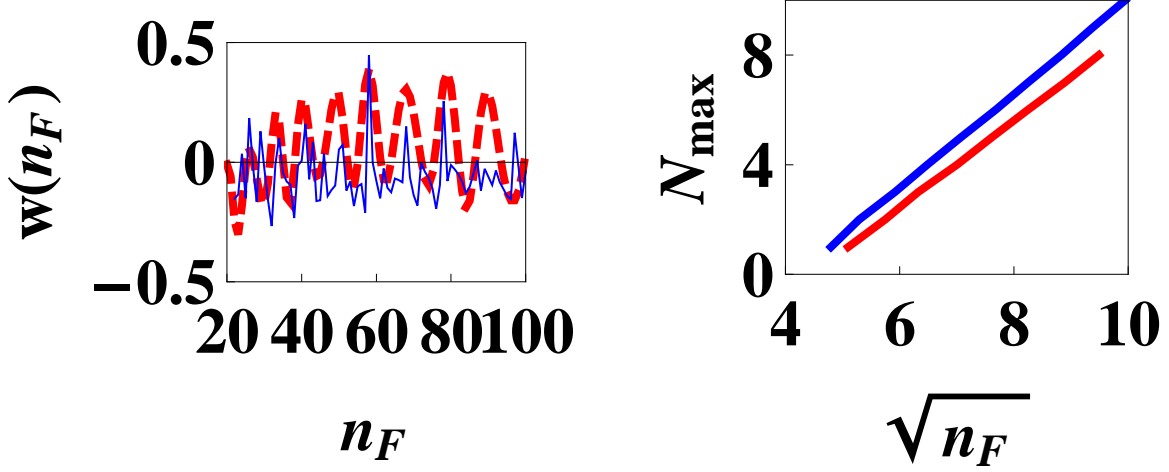


FIG. 1: (color online): Left panel: The "erratic" function $w(n_F)$ (solid blue line) plotted together with the characteristic smooth function $\Xi(n_F)$ (dashed red line) in the interval $20 \leq n_F \leq 100$. Right panel: Number of maxima, $N_{\max}(\sqrt{n_F})$, of the characteristic function $\Xi(n_F)$ for a square vortex lattice (red line) and for a hexagonal vortex lattice (blue line).

$$\beta_{\mathbf{G}}(\gamma) = \frac{1}{2} \left\{ \frac{1}{(1+\gamma)} \exp \left[- \left(\frac{1-\gamma}{1+\gamma} \right) |\mathbf{G}|^2 \right] + \frac{1}{(1-\gamma)} \exp \left[- \left(\frac{1+\gamma}{1-\gamma} \right) |\mathbf{G}|^2 \right] \right\} \quad (3)$$

where \mathbf{G} is a reciprocal vortex lattice vector. The structure function $\beta(\gamma)$ controls the coupling between the two pairs of electrons involved and the vortex lattice, where γ is a periodic function of the individual electronic "time" variables τ_j , which reflects the underlying electron cyclotron motions. Note that $\beta(\gamma=0)$ is the well known Abrikosov parameter β_A for an arbitrary vortex lattice geometry⁷. The most remarkable feature of $\beta(\gamma)$ seen in Eq.(3) is associated with the dual singular points at $\gamma \rightarrow \pm 1$, where τ_j , approach, respectively: $\tau_1 = \tau_3 \rightarrow 0, \tau_2 \rightarrow n\pi - \tau, \tau_4 \rightarrow n\pi + \tau$, or: $\tau_1 \rightarrow n\pi - \tau, \tau_3 \rightarrow n\pi + \tau, \tau_2 = \tau_4 \rightarrow 0$, with τ being an arbitrary real number in the interval: $-\pi \leq \tau \leq \pi$, and $n = 1, 2, \dots$. The electrons at such highly correlated pairs of cyclotron orbits are singularly scattered by the vortex lattice, yielding only purely harmonic contributions to the SC free energy in the dHvA frequency $F = n_F H$ since under these conditions: $e^{-in_F(\tau_1 - \tau_2 + \tau_3 - \tau_4)} \rightarrow e^{-2\pi i n n_F}$. The pair of electrons whose positions on the singular cyclotron orbit coincide (i.e. (1,3) at $\gamma \rightarrow 1$ and (2,4) at $\gamma \rightarrow -1$) undergo local mutual scattering and so exchange many G -vectors through the vortex lattice during the scattering process, while those electrons moving coherently on a large cyclotron orbit in opposite directions (i.e. (1,3) at $\gamma \rightarrow -1$ and (2,4) at $\gamma \rightarrow 1$) are mutually scattered over a long range of the vortex lattice, and so do not exchange momentum. Thus, the leading purely harmonic contributions to $\beta(\gamma)$ are: $\beta_{\mathbf{G}=0}(\gamma) = \frac{1}{2} \left(\frac{1}{1+\gamma} + \frac{1}{1-\gamma} \right)$, and $\int \beta_{\mathbf{G}}(\gamma) d^2 G$, which yields: $\frac{1}{2} \frac{1}{1-\gamma}$ for $\gamma \rightarrow 1$, and $\frac{1}{1+\gamma}$ for $\gamma \rightarrow -1$.

Significant deviations from the harmonic part are developed slightly away from the singular points. They can be estimated by expanding $(1-\gamma)/(1+\gamma)$ about the singular points, e.g. for $\gamma = 1$: $(1-\gamma)/(1+\gamma) \simeq -i\tilde{\xi}_2/4 + (4\tilde{\xi}_1^2 + \tilde{\xi}_3^2)/16$, with $\tilde{\xi}_1 = (\tau_1 + \tau_2 + \tau_3 + \tau_4)/4 - \pi/2$, $\tilde{\xi}_2 = (\tau_1 - \tau_2 + \tau_3 - \tau_4)/2 + \pi$, $\tilde{\xi}_3 = \tau_1 - \tau_3$, and carrying out the τ_j -integrations. Focusing, for simplicity, on the deviations from the first harmonic it is found that: $I_4 \rightarrow \text{Re} e^{2\pi i n_F} e^{-2\pi i \varpi_\nu} \int_0^\infty d\tilde{\xi}_1 e^{-4\varpi_\nu \tilde{\xi}_1} I_3(\tilde{\xi}_1)$, where:

$$\begin{aligned} I_3(\tilde{\xi}_1) = & 2\text{Re} \sum_{\mathbf{G}} \int_{-2\tilde{\xi}_1}^{2\tilde{\xi}_1} (2\tilde{\xi}_1 - \tilde{\xi}_2) d\tilde{\xi}_2 \exp \left\{ i\tilde{\xi}_2 \left[\frac{1}{4} |\mathbf{G}|^2 - 2n_F \right] \right\} \\ & \times \int_{-(2\tilde{\xi}_1 + \tilde{\xi}_2)}^{2\tilde{\xi}_1 + \tilde{\xi}_2} d\tilde{\xi}_3 \exp \left[- \left(\tilde{\xi}_1^2 + \frac{1}{4} \tilde{\xi}_3^2 \right) \frac{1}{4} |\mathbf{G}|^2 \right] \end{aligned} \quad (4)$$

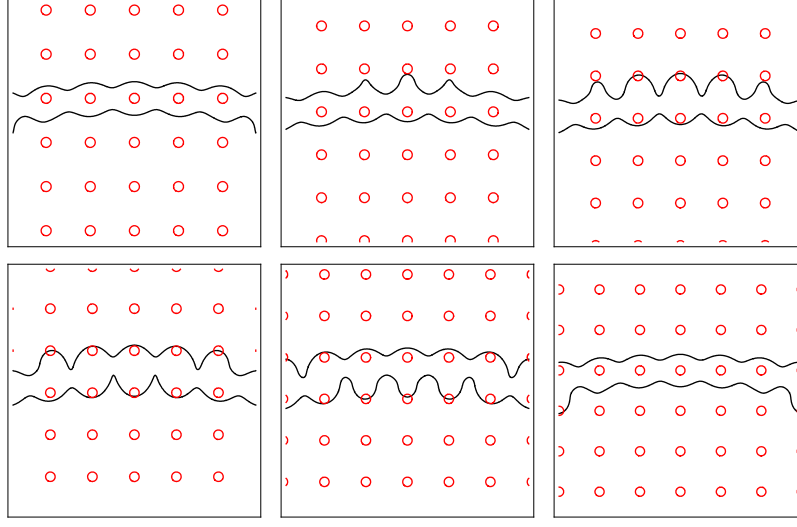


FIG. 2: (color online): A square lattice of vortex cores (small red circles), shown sweeping across a pair of classical orbits occupied by a quasi particle-quasi hole pair (upper and lower solid curves respectively) in a magnetic field whose magnitude increases successively from the upper left panel through the lower right panel. Note that in the ground state the occupied (low-energy branch) cyclotron orbit is locally distorted paramagnetically in passing through vortex core regions, whereas both orbital branches of a quasi particle-hole excitation are distorted diamagnetically.

satisfying: $|\mathbf{G}| \approx 2\sqrt{2n_F}$, namely having length close to the Fermi surface diameter. Furthermore, due to the large values of n_F and the discrete nature of \mathbf{G} (which are measured in units of a_H^{-1}), the integration over ξ_2 yields "erratically" oscillating, quasi-periodic function of $\sqrt{n_F}$, superimposed on the usual dHvA oscillation, expressed by the factor $e^{2\pi i n_F}$ of I_4 in our simple spherical Fermi surface model. As will be illustrated below, the corresponding oscillating envelop function contains vortex-lattice structural information which can be extracted from experiment by an appropriate data analysis (see Fig.1).

The final result for the first dHvA harmonic of the SC TP, up to fourth order, can be written in the form:

$$\Omega_{sc}^{(1h)}/\Omega_n^{(1h)} \simeq 1 - \frac{\pi^{3/2}}{\sqrt{n_F}} \left| \frac{\Delta_0}{\hbar\omega_c} \right|^2 + \frac{1}{2} (1 + w(n_F)) \frac{\pi^3}{n_F} \left| \frac{\Delta_0}{\hbar\omega_c} \right|^4 \quad (5)$$

where $\Omega_n^{(1h)}$ is the corresponding normal state free energy extrapolated to the vortex state, and $w(n_F)$, shown in Fig.1, represents the "erratically" oscillating component.

The influence of vortex-lattice disorder on the SC free energy in the white noise limit can be determined by invoking the general expansion of the state function $\varphi_0(x, y)$ in terms of Landau orbitals wave functions with randomly selected coefficients. Averaging $\beta(\gamma)$ over realizations of these coefficients it can be easily shown that: $\langle \beta(\gamma) \rangle \rightarrow \frac{1}{1-\gamma} + \frac{1}{1+\gamma}$, which is just the purely harmonic component of $\beta(\gamma)$. In this limiting case, only incoherent scattering processes by the vortex matter contribute to the SC TP, and the final result, up to fourth order, is obtained from Eq.(5) by taking $w(n_F) \rightarrow 0$, i.e. very

close to the well known Maki expression⁹, as expanded to the same power in Δ_0 .

It is interesting to note that the kinematical condition, $|\mathbf{G}| = 2\sqrt{2n_F}$, is equivalent to the real space condition, $|\mathbf{R}| = 2\sqrt{2n_F}$ (where \mathbf{R} is a vortex-lattice vector measured in units a_H), which is just the condition for the cyclotron orbit at the Fermi energy to pass through a vortex core. The "erratic" oscillations are closely related to such cyclotron orbits since the latter are strongly distorted during their passage near a vortex core⁷. The influence of a "sweeping" vortex lattice (generated by a sweeping magnetic field) crossing a quasi-particle quasi-hole pair of cyclotron orbits may be studied qualitatively by considering a highly simplified model of a classical charged particle moving in two dimensions under a perpendicular magnetic field $\mathbf{H} = H\hat{z}$ and in the presence of a "pair-potential", $|\Delta(\mathbf{r})|$, with the Bogoliubov dispersion relation $\epsilon(\mathbf{k}) = \pm \sqrt{\xi_k^2 + |\Delta(\mathbf{r})|^2}$, where $\xi_k = \frac{\hbar^2 k^2}{2m^*} - E_F$ ⁷. An important advantage of this model, despite its obvious shortcomings, is that it can be implemented without invoking perturbation theory with respect to $\Delta(\mathbf{r})$. The corresponding coupled Lorentz-Bloch velocity equations:

$$\frac{d\mathbf{k}}{dt} = - \left[\frac{d\mathbf{r}}{dt} \times \hat{z} \right] ; \quad \frac{d\mathbf{r}}{dt} = \frac{1}{\hbar} \frac{d}{d\mathbf{k}} \epsilon(\mathbf{k}) .$$

can be easily integrated to yield the trajectory equation:

$$\left[r^2 - (k_F/h)^2 \right]^2 + 4 \left| \frac{\Delta(r, \phi)}{\hbar\omega_c \hbar^2} \right|^2 = r_0^4$$

where $h = H/H_{c2}$ and r_0 is an integral of motion determined by initial conditions. Obviously, there are two

types of solutions for energies above and below the Fermi energy, corresponding to the Bogoliubov particle-hole pair. Examples of classically allowed, distorted cyclotron trajectories, which correspond to real solutions $r(\phi)$, are depicted in Fig.2. They are restricted to QP energies above the threshold $r_0^2 = 2 \max |\Delta(r, \phi)| / \hbar \omega_{c2} h^2$. Below this threshold there are complex solutions corresponding to tunneling of QP orbits between vortex cores (not shown). For increasing magnetic field strength h at a fixed QP energy, the lattice of vortex cores, shown in Fig.2 successively from the upper-left panel through the lower-right panel, sweeps across the QP cyclotron orbits, which undergo a strong local distortion upon crossing a vortex core. The appearances of such local distortions are closely related to the "erratic" paramagnetic-diamagnetic oscillations of the function $w(n_F)$ shown in Fig.1. A characteristic feature of $w(n_F)$ to be exploited for distinguishing between different vortex-lattice structures can be conveniently defined in terms of the smooth function $\Xi(n_F) = a \sum_{\mathbf{G}} \exp \left[-\frac{1}{4} |\mathbf{G}|^2 \left(1 - 8n_F / |\mathbf{G}|^2 \right)^2 \right] - b$ (see

Fig.1, left panel), derived with the help of Eq.(4) by performing the integral $\int_0^\infty d\tilde{\xi}_1 e^{-4\varpi_\nu \tilde{\xi}_1} I_3(\tilde{\xi}_1)$ in the stationary phase approximation. Here a and b are adjustable parameters. The number of maxima, $N_{\max}(\sqrt{n_F})$ of $\Xi(n_F)$ appearing below a given value of n_F offers the desired characteristic feature (see Fig.1, right panel). Its slope is a measure of the rate at which vortex cores in Fig.2 cross the cyclotron orbit at the Fermi energy, and so uniquely characterizes the point symmetry of the underlying vortex lattice.

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